

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Question Set 3

1 In this question you must show detailed reasoning.

Solve the equation $x(3 - \sqrt{5}) = 24$, giving your answer in the form $a + b\sqrt{5}$, where a and b are positive integers. [3]

$$x = \frac{24}{3 - \sqrt{5}} = \frac{24}{3 - \sqrt{5}} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{72 + 24\sqrt{5}}{4} = 18 + 6\sqrt{5}$$

2 (a) Express $5x^2 - 20x + 3$ in the form $p(x+q)^2 + r$, where p , q and r are integers. [3]

$$5(x-2)^2 - 17$$

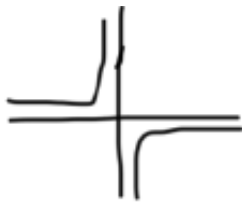
(b) State the coordinates of the minimum point of the curve $y = 5x^2 - 20x + 3$. [2]

$$(2, -17)$$

(c) State the equation of the normal to the curve $y = 5x^2 - 20x + 3$ at its minimum point. [1]

$$x = 2$$

3 (a) Sketch the curve $y = -\frac{1}{x^2}$. [1]



(b) The curve $y = -\frac{1}{x^2}$ is translated by 2 units in the positive x -direction.

State the equation of the curve after it has been translated. [2]

$$y = -\frac{1}{(x-2)^2}$$

(c) The curve $y = -\frac{1}{x^2}$ is stretched parallel to the y -axis with scale factor $\frac{1}{2}$ and, as a result, the point $(\frac{1}{2}, -4)$ on the curve is transformed to the point P .

State the coordinates of P . [2]

$$\left(\frac{1}{2}, -2\right)$$

- 4 (a) Find and simplify the first three terms in the expansion of $(2 - 5x)^5$ in ascending powers of x . [3]

$$32 + {}^5C_1 \times 2^4 \times (-5x) + {}^5C_2 \times 2^3 \times (-5)^2 x^2$$

$$= 32 - 400x + 2000x^2$$

- (b) In the expansion of $(1 + ax)^2(2 - 5x)^5$, the coefficient of x is 48.

Find the value of a . [3]

$$(a^2x^2 + 2ax + 1)(32 - 400x + \dots) = 32 - 400x + 64ax$$

$$-400x + 64ax = 48x \quad a = 7$$

- 5 Points A, B, C and D have position vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} 4 \\ k \end{pmatrix}$.

- (a) Find the value of k for which D is the midpoint of AC . [1]

3

- (b) Find the two values of k for which $|\overrightarrow{AD}| = \sqrt{13}$. [3]

$$(4-1)^2 + (k-2)^2 = 13$$

$$(k-2)^2 = 4 \quad k-2 = \pm 2 \quad k=0 \quad k=4$$

- (c) Find one value of k for which the four points form a trapezium. [2]

$$\overrightarrow{BD} = x \overrightarrow{AC}$$

$$\begin{pmatrix} 1 \\ k-5 \end{pmatrix} = x \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$x = \frac{1}{6}$$

$$(k-5) = \frac{1}{3}$$

$$k = \frac{16}{3}$$

6 In this question you must show detailed reasoning.

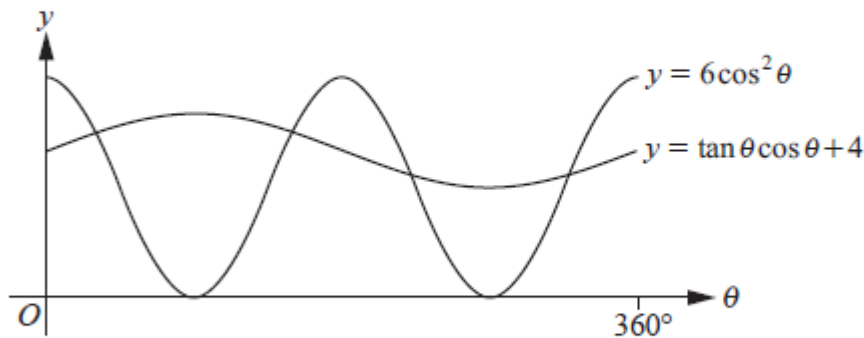
(a) Show that the equation $6 \cos^2 \theta = \tan \theta \cos \theta + 4$

can be expressed in the form $6 \sin^2 \theta + \sin \theta - 2 = 0$.

[2]

$$\begin{aligned}
 6(1 - \sin^2 \theta) &= \sin \theta + 4 \\
 6 - 6\sin^2 \theta &= \sin \theta + 4 \\
 6\sin^2 \theta - \sin \theta - 2 &= 0
 \end{aligned}$$

(b)



The diagram shows parts of the curves $y = 6 \cos^2 \theta$ and $y = \tan \theta \cos \theta + 4$, where θ is in degrees.

Solve the inequality $6 \cos^2 \theta > \tan \theta \cos \theta + 4$ for $0^\circ < \theta < 360^\circ$.

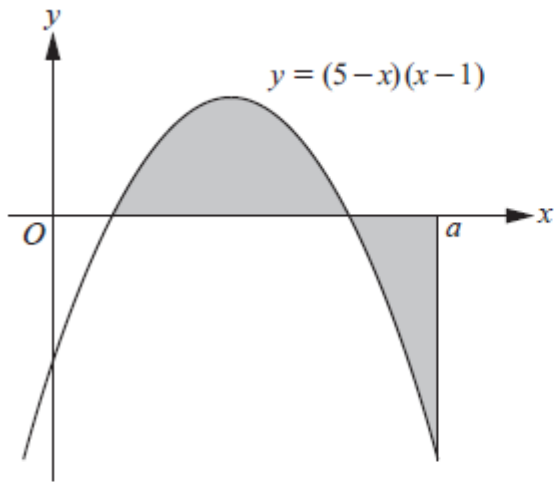
[5]

$$\begin{aligned}
 6 \sin^2 \theta - \sin \theta - 2 &< 0 \\
 (3 \sin \theta - 2)(2 \sin \theta + 1) & \\
 \sin \theta < \frac{2}{3} \quad \sin \theta > -\frac{1}{2} & \\
 \arcsin \frac{2}{3} = 41.8 \quad \arcsin -\frac{1}{2} = \frac{210}{330} & \\
 138.2 &
 \end{aligned}$$

Looking at graph

$$\begin{aligned}
 \text{true when: } & 0 < \theta < 41.8 \\
 & 138.2 < \theta < 210 \\
 & 330 < \theta < 360
 \end{aligned}$$

7



The diagram shows part of the curve $y = (5-x)(x-1)$ and the line $x = a$.

Given that the total area of the regions shaded in the diagram is 19 units^2 , determine the exact value of a . [8]

area below x-axis -ve

$$\int_1^5 (5-x)(x-1) dx = \int_5^a (5-x)(x-1) dx$$

$$19 = \left[-\frac{1}{2}x^2 + 3x^2 - 5x \right]_1^5 - \left[\frac{1}{2}x^2 + 3x^2 - 5x \right]_5^a$$

$$19 - \left(\frac{25}{2} - \frac{7}{2} \right) = \frac{32}{2} - \left(-\frac{a^2}{2} + 3a^2 - 5a \right) - \frac{25}{2}$$

$$19 - \frac{32}{2} = \frac{25}{2} - \frac{a^2}{2} + 6a - 5$$

$$\frac{a^3}{3} - 3a^2 + 5a = 0 \text{ as } \frac{25}{3} = \left(\frac{25}{3} + \frac{a^3}{3} + 3a^2 - 5a \right) a^2 + 9a - 15 = 0$$

$$a = \frac{9 + \sqrt{21}}{2}$$

8

(a) Show that the equation $2 \log_2 x = \log_2(kx-1) + 3$, where k is a constant, can be expressed in the form $x^2 - 8kx + 8 = 0$. [4]

$$\log_2 x^2 = \log_2(8kx-8) \quad x^2 = 8kx - 8 \quad x^2 - 8kx + 8 = 0 \quad \log_2 8 + \log_2(kx-1) = \log_2(8kx-8)$$

(b) Given that the equation $2 \log_2 x = \log_2(kx-1) + 3$ has only one real root, find the value of this root. [4]

$$\frac{8k + \sqrt{64k^2 - 32}}{2} \quad \frac{8k + \sqrt{64k^2 - 32}}{2} \text{ as root needs to be positive}$$

$$x = 4k + 2\sqrt{4k^2 - 2}$$

Total Marks for Question Set 3: 49

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