AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Question Set 3

1 **In this question you must show detailed reasoning.**

Solve the equation $x(3-\sqrt{5}) = 24$, giving your answer in the form $a + b\sqrt{5}$, where a and b are positive integers. **[3]**

$x = \frac{24}{3-\sqrt{5}}$ $\frac{24}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{72+24\sqrt{5}}{4} = 18+6\sqrt{5}$

2 **(a)** Express $5x^2 - 20x + 3$ in the form $p(x+q)^2 + r$, where p, q and r are integers. **[3]**

$5(x-2)^2 - 17$

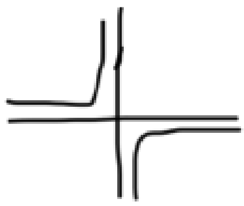
(b) State the coordinates of the minimum point of the curve $y = 5x^2 - 20x + 3$. **[2]**

$(2, -17)$

(c) State the equation of the normal to the curve $y = 5x^2 - 20x + 3$ at its minimum point. **[1]**

$x = 2$

3 **(a)** Sketch the curve $y = -\frac{1}{x^2}$. **[1]**



(b) The curve $y = -\frac{1}{x^2}$ is translated by 2 units in the positive x-direction.

State the equation of the curve after it has been translated. **[2]**

$y = \frac{-1}{(x-2)^2}$

(c) The curve $y = -\frac{1}{x^2}$ is stretched parallel to the y-axis with scale factor $\frac{1}{2}$ and, as a result, the point $\left(\frac{1}{2}, -4\right)$ on the curve is transformed to the point P.

State the coordinates of P. **[2]**

$\left(\frac{1}{2}, -2\right)$

4 **(a)** Find and simplify the first three terms in the expansion of $(2-5x)^5$ in ascending powers of x. **[3]**

$32 + {}^5C_1 \times 2^4 x - 5x + {}^5C_2 \times 2^3 \times (-5x)^2$

$= 32 - 400x + 2000x^2$

(b) In the expansion of $(1+ax)^2(2-5x)^5$, the coefficient of x is 48.

Find the value of a. **[3]**

$(a^2x^2 + 2ax + 1)(32 - 400x \ldots) = 32 - 400x + 64ax$

$-400x + 64ax = 48x \quad a = 7$

5

Points A, B, C and D have position vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} 4 \\ k \end{pmatrix}$.

(a) Find the value of k for which D is the midpoint of AC. **[1]**

3

(b) Find the two values of k for which $|\overrightarrow{AD}| = \sqrt{13}$. **[3]**

$(4-1)^2 + (k-2)^2 = 13$

$(k-2)^2 = 4 \quad k-2 = \pm 2 \quad k=0 \quad k=4$

(c) Find one value of k for which the four points form a trapezium. **[2]**

$\overrightarrow{BD} = x\overrightarrow{AC}$

$\begin{pmatrix} 1 \\ k-5 \end{pmatrix} = x\begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$x = \frac{1}{6}$

$(k-5) = \frac{1}{3}$

$k = \frac{16}{3}$

6 **In this question you must show detailed reasoning.**

(a) Show that the equation $6\cos^2\theta = \tan\theta\cos\theta + 4$

$\frac{\sin\theta}{\cos\theta}$

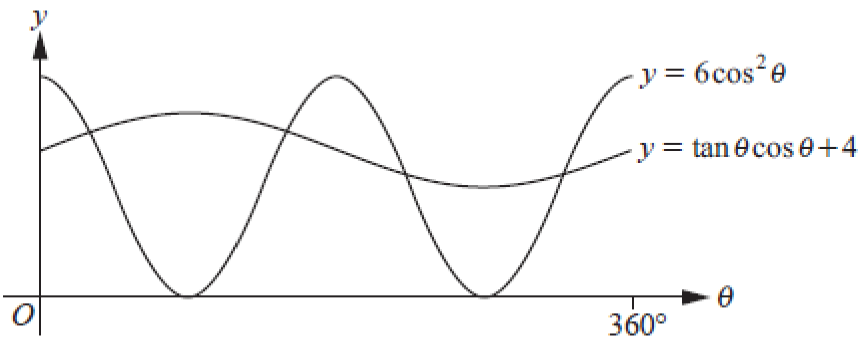
can be expressed in the form $6\sin^2\theta + \sin\theta - 2 = 0$. **[2]**

$6(1-\sin^2\theta) = \sin\theta + 4$

$6 - 6\sin^2\theta = \sin\theta + 4$

$6\sin^2\theta - \sin\theta - 2 = 0$

(b)



The diagram shows parts of the curves $y = 6\cos^2\theta$ and $y = \tan\theta\cos\theta + 4$, where θ is in degrees.

Solve the inequality $6\cos^2\theta > \tan\theta\cos\theta + 4$ for $0° < \theta < 360°$. **[5]**

$6\sin^2\theta - \sin\theta - 2 < 0$

$(3\sin\theta - 2)(2\sin\theta + 1)$

$\sin\theta < \frac{2}{3}$ $\quad$ $\sin\theta > -\frac{1}{2}$

$\arcsin\frac{2}{3} = 41.8$ $\quad$ $\arcsin -\frac{1}{2} = 210$, 330

138.2

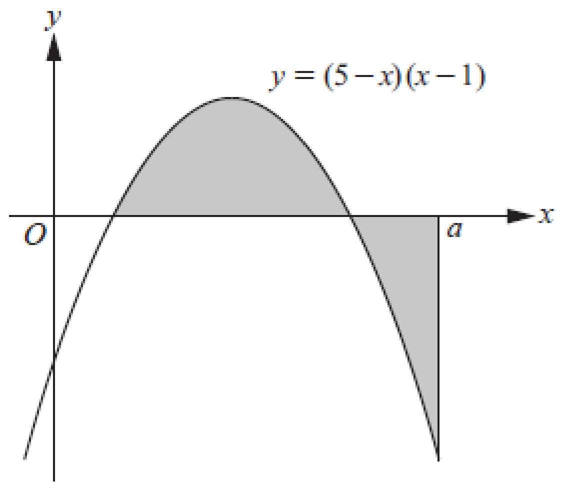
Looking at graph true when:

$0 < \theta < 41.8$

$138.2 < \theta < 210$

$330 < \theta < 360$

7



The diagram shows part of the curve $y = (5-x)(x-1)$ and the line $x = a$.

Given that the total area of the regions shaded in the diagram is 19 units^2, determine the exact value of a. **[8]**

area below x axisis -ve

$\int_1^5 (5-x)(x-1)\,dx - \int_5^a (5-x)(x-1)\,dx$

$19 = [-\frac{1}{3}x^3+3x^2-5x]_1^5 - [-\frac{1}{3}x^3+3x^2-5x]_5^a$

$19 - (\frac{25}{3} - -\frac{7}{3}) = \frac{32}{3}$ $\quad -((-\frac{a^3}{3}+3a^2-5a)-\frac{25}{3})$

$-x^2+6x-5$

$19-\frac{32}{3} = \frac{25}{3}$ $\quad \frac{a^3}{3}-3a^2+5a=0$ as $\frac{25}{3} = -(\frac{25}{3}+\frac{a^3}{3}+3a^2-5a)$ $\quad a^2+9a-15=0$

$a = \frac{9+\sqrt{21}}{2}$

8

(a) Show that the equation $2\log_2 x = \log_2(kx-1)+3$, where k is a constant, can be expressed in the form $x^2 - 8kx + 8 = 0$. **[4]**

$\log_2 x^2 = \log_2(8kx-8)$ $\quad x^2 = 8kx-8$ $\quad x^2-8kx+8=0$ | $\log_2 8 + \log_2(kx-1) = \log_2(8kx-8)$

(b) Given that the equation $2\log_2 x = \log_2(kx-1)+3$ has only one real root, find the value of this root. **[4]**

$\frac{8k \pm \sqrt{64k^2-32}}{2}$ $\quad \frac{8k+\sqrt{64k^2-32}}{2}$ as root needs to be positive

$x = 4k+2\sqrt{4k^2-2}$

Total Marks for Question Set 3: 49